Student's Name:_____ Teacher's Code : _____

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2021

Year 12
Mathematics
Task 4
Trial HSC Replacement Task
Date : 27th August 2021

General Instructions:

- **Reading time : 5 minutes**
- **Time Allowed: 2 hours**
- Write using blue or black pen only
- NESA approved calculators may be used
- Attempt all questions in the space provided on the paper
- Write **your name** and **your teacher's code** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working.

Teachers :

- Mr N Mushan **NHM**
- Mr P Collins **PPC**
- Dr M Furtado **MXF**
- Mrs F Yates **FEY**
- Mr J Newey **JPN**

Format:

Section A Short Answer	**14 Marks**
Section B Short Answer	**15 Marks**
Section C Short Answer	**16 Marks**
Section D Short Answer	**11 Marks**
Section E Short Answer	**14 Marks**
Total	**70 Marks**

SECTION A (14 Marks) (Start a new booklet)

(a) Find the equation of the tangent to the curve $y=(1-5x)^{-1}$ at the point (0, 1). **(3)**

(b) Differentiate $y=\dfrac{\ln\left(x^3\right)}{2}$ with respect to x. **(2)**

(c) Differentiate $y=\dfrac{e^{2x}}{\cos x}$ with respect to x. **(2)**

(d) Evaluate $\int_0^{\frac{\pi}{3}}\left(x+\sin x\right)dx$. **(2)**

(e) The function $y=f(x)$ is continuous for all values of x. **(2)**

The following is known of the function's properties.

- $x>-2,\ f'(x)<0,\ f''(x)<0$
- $x<-2,\ f'(x)<0,\ f''(x)>0$
- $f(-2)\ =\ f'(-2)\ =\ f''(-2)=0$

Sketch the graph of $y=f(x)$, labelling any x-intercepts **in your own booklet**.

(f) Solve the equation $\ln x\ -\ \dfrac{6}{\ln x}=1$ **(3)**

END OF SECTION A

SECTION B (15 Marks) (Start a new booklet)

(a) An infinite geometric series has a first term of 12 and a limiting sum of 15. **(2)**

What is the common ratio?

(b) A function $f(x)$ is given by:

$$f(x)=\begin{cases}\frac{3}{4}(x-2)(4-x) & \text{if } 2\le x\le 4\\ 0 & \text{if } x<2 \text{ or } x>4\end{cases}$$

(i) Show this curve is a probability density function. **(2)**

(ii) Find the Mode. **(2)**

(c) (i) Draw the graph of $y=-\sqrt{4-x^2}$ and shade the area bounded by the function and the x axis where $x\le 0$. **(2)**

The table gives the values of $y=-\sqrt{4-x^2}$ for x.

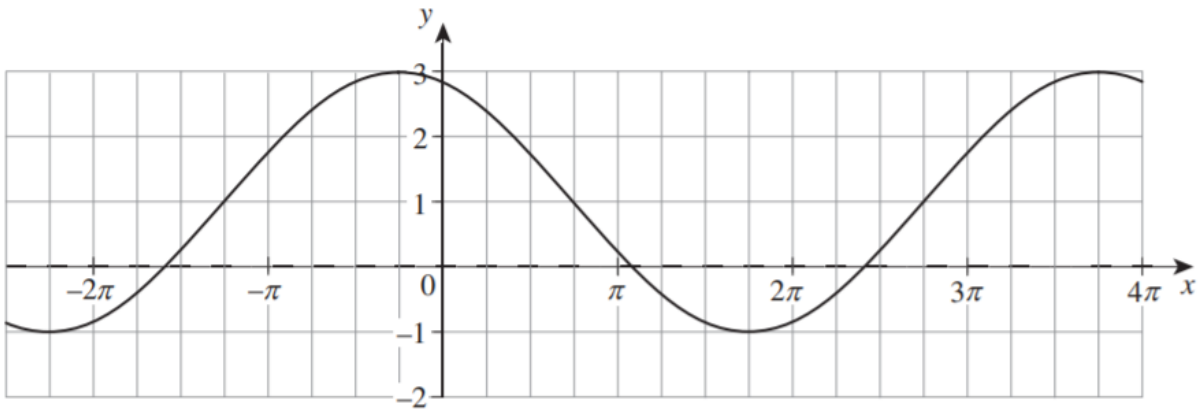
x	-2	-1.5	-1	-0.5	0
y	0	-1.3	-1.7	-1.9	-2

(ii) By using the above table of values or otherwise, use the trapezoidal rule with 4 sub-intervals to find an approximation of the shaded area. **(2)**

(iii) How could a better approximation of the shaded area be obtained using the trapezoidal rule? **(1)**

SECTION B is continued on next page

(d) The graph is a function of the form $y = k\sin(a(x+b))+c$.



Consider the equation of the function, to determine the values of k, a, b and c **(4)**

END OF SECTION B

SECTION C (16 Marks) (Start a new booklet)

(a) Let X be a normally distributed random variable with mean 6 and variance 9 and let Z be the random variable with the standard normal distribution.

(i) Find $P(X > 6)$. **(1)**

(ii) Find b such that $P(X > 8) = P(Z < b)$. **(2)**

(b) For events A and B from a sample space, $P(A|B) = \frac{1}{5}$ and $P(B|A) = \frac{1}{4}$.

Let $P(A \cap B) = p$.

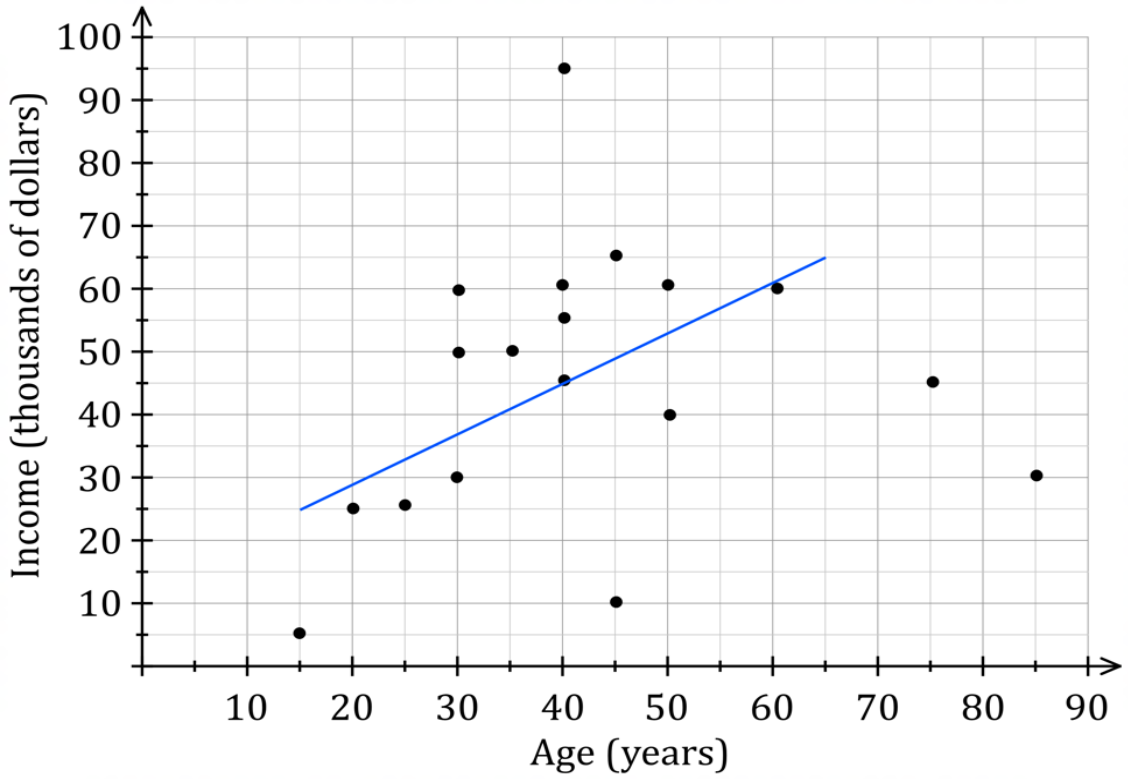
(i) Find $P(A)$ in terms of p. **(1)**

(ii) Find $P(A' \cap B')$ in terms of p. **(2)**

(iii) Given that $P(A \cup B) \leq \frac{1}{5}$, state the largest possible interval for p. **(2)**

SECTION C is continued on next page

(c) A sample of 18 randomly selected people were asked to state their Age (in years) and their income (in $1000). The results are displayed in the scatterplot below with the line of best fit drawn through (15, 25) and (65, 65).



(i) Calculate the gradient of the line of best fit shown. **(2)**

(ii) What is the equation of the line of best fit shown? **(1)**

(iii) Use your equation to calculate the expected income of an 80-year-old person. **(1)**

(iv) Comment on your result in part (iii). **(1)**

(d) (i) Without using calculus, sketch $y = \ln x$ in your booklet. **(1)**

(ii) On the same sketch in part (a), find, graphically, the number of solutions to the equation $\ln x - x = -2$. State the number of solutions in your booklet. **(2)**

END OF SECTION C

SECTION D (11 Marks) (Start a new booklet)

(a) A chemical substance being made in a laboratory will be airlifted by helicopter to a hospital immediately after it is completed. The substance decomposes and the amount M in kilograms present at any time t hours is given by $M = Ae^{kt}$ where A and k are constants.

If $\frac{1}{4}$ of the mass of this substance is present after 4 hours and 4 kg of this substance will reach the hospital in 6 hours.

(i) Show the exact value of $k = -\frac{1}{4}\ln(4)$ **(2)**

(ii) Find the value of A, the original mass of the chemical substance. **(2)**

(b) An open cylindrical water tank has base radius x metres and height h metres.

Each square metre of the base costs a dollars to manufacture and each square metre of the curved surface costs b dollars, where a and b are constants.

The combined cost of the base and curved surface is c dollars.

(i) Find c in terms of a, b, x and h. **(1)**
(Note that the curved surface has area $2\pi xh$.)

(ii) Show that the volume V of the tank in cubic metres is given by **(2)**

$$V = \frac{x}{2b}(c - \pi a x^2)$$

(iii) If x can vary, prove that V is maximised when the **cost of the base** is $\frac{c}{3}$ dollars. **(4)**

END OF SECTION D

SECTION E (14 Marks) (Start a new booklet)

(a) Alison decides to start saving for a deposit for a new apartment by investing \$1000 into a bank account at the start of every month. Interest is paid at a rate of 3% per annum compounded monthly.

(i) Show that the amount in A_n in the account after n months is: **(2)**

$$A_n = 401000(1.0025^n - 1)$$

(ii) Calculate the amount in the account at the end of 12 months. **(1)**
Give your answer to the nearest dollar.

(iii) Determine how many deposits in total Alison will have made when her account first exceeds \$60 000. **(2)**

(iv) After twelve months Alison changes her contributions to \$1500 per month. **(2)**
Determine if Alison will have saved \$60 000 by the end of the third year.
Justify your answer.

(b) (i) Show that $x = \frac{\pi}{3}$ is a solution of $\sin x = \frac{1}{2}\tan x$ **(1)**

(ii) On the same set of axes, sketch the graphs of the functions **(2)**

$y = \sin x$ and $y = \frac{1}{2}\tan x$ for $-\pi \le x \le \pi$.

(iii) Hence find all solutions of $\sin x = \frac{1}{2}\tan x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **(2)**

(iv) Use your graphs to solve $\sin x \le \frac{1}{2}\tan x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **(2)**

END OF PAPER

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SUGGESTED SOLUTIONS

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SECTION A

Worked Solution		Marker's Comments
ⓐ $y = (1-5x)^{-1}$		
$y' = -1(1-5x)^{-2} \times -5$		
$= 5(1-5x)^{-2}$ or $\frac{5}{(1-5x)^2}$	✓	1 mark for differentiating
When $x=0$		
$m_{tangent} = \frac{5}{(1-5\times0)^2} = 5$	✓	1 mark for gradient of tangent
equation of tangent		
$y-1 = 5(x-0)$		
$y-1 = 5x$	✓	
$\boxed{y = 5x+1}$ or $5x-y+1=0$		1 mark for equation of tangent.
		reasonably well done
ⓑ $y = \frac{\ln(x^3)}{2}$		Alternative Solution
$= \frac{1}{2}\ln(x^3)$		$y = \frac{3\ln x}{2}$ ✓
$y' = \frac{1}{2} \times \frac{3x^2}{x^3}$	✓	$y' = \frac{3}{2} \times \frac{1}{x}$
$= \frac{3x^2}{2x^3}$		$y' = \frac{3}{2x}$ ✓
$= \boxed{\frac{3}{2x}}$	✓	many failed to simplify expression. (∴ lost a mark).
ⓒ $y = \frac{e^{2x}}{\cos x}$		
let $u = e^{2x}$ $\quad v = \cos x$		
$u' = 2e^{2x}$ $\quad v' = -\sin x$	✓	1 mark for correctly finding u' and v'.
$y' = \frac{2e^{2x}\cos x + e^{2x}\sin x}{\cos^2 x}$	✓	correctly substituting into product rule.
$y' = \boxed{\frac{e^{2x}(2\cos x + \sin x)}{\cos^2 x}}$		- some forgot v^2 - some mixed up signs.

Section A (continued)

Marker's Comments

(d) $\int_0^{\frac{\pi}{3}} (x + \sin x)\,dx$

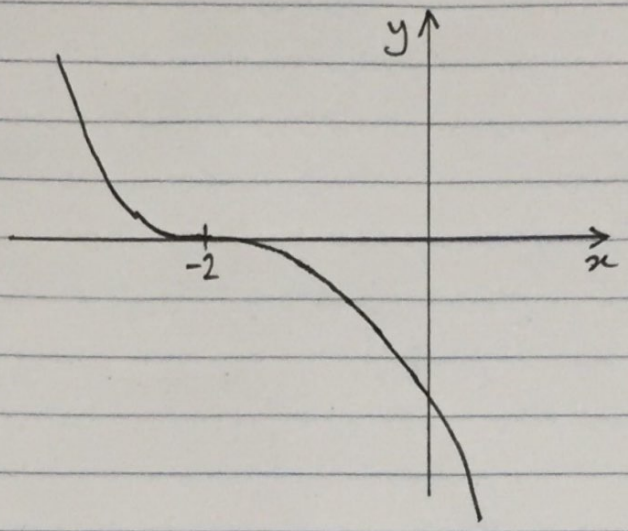
$= \left[\frac{x^2}{2} - \cos x\right]_0^{\frac{\pi}{3}}$ ✓ integrating.

$= \left(\frac{\left(\frac{\pi}{3}\right)^2}{2} - \cos\frac{\pi}{3}\right) - \left(\frac{0^2}{2} - \cos 0\right)$

$= \frac{\pi^2}{18} - \frac{1}{2} + 1$ many messed up the arithmetic.

$= \boxed{\frac{\pi^2}{18} + \frac{1}{2}}$ or $\frac{\pi^2+9}{18}$ ✓

(e)



- had to show that there is a stationary point at $x=-2$ $(-2,0)$
- concave up to left of $x=-2$
- concave down to right of $x=-2$

✓ correct shape

✓ clearly showing horizontal point of inflection at $x=-2$ ie it must be obvious.

(f) $\ln x - \frac{6}{\ln x} = 1$

$(\ln x)^2 - 6 = \ln x$

$(\ln x)^2 - \ln x - 6 = 0$ ✓ 1 mark for multiplying through by x and forming correct quadratic. (note $(\ln x)^2 \neq \ln(x^2)$)

$(\ln x - 3)(\ln x + 2) = 0$

$\ln x = 3$ or -2 ✓ solving quadratic

ie $\log_e x = 3$ or $\log_e x = -2$

$\therefore x = e^3$ or $x = e^{-2}$ ✓ two correct solutions.

Section B

Solutions & Comments

a) $a = 12$

$S_\infty = 15$

$S_\infty = \frac{a}{1-r}$

$15 = \frac{12}{1-r}$

$15 - 15r = 12$

$-15r = -3$

$r = \frac{1}{5}$

* well done by all

b) Proving a PDF

1) Prove $\int_2^4 f(x)\,dx = 1$

$$\int_2^4 \frac{3}{4}(x-2)(4-x)\,dx$$

$$= \frac{3}{4}\int_2^4 -x^2 + 6x - 8\,dx$$

$$= \frac{3}{4}\left[-\frac{x^3}{3} + 3x^2 - 8x\right]_2^4$$

$$= \frac{3}{4}\left[\left(-\frac{64}{3} + 48 - 32\right) - \left(-\frac{8}{3} + 12 - 16\right)\right]$$

$$= \frac{3}{4}\left[-\frac{16}{3} + \frac{20}{3}\right]$$

$$= \frac{3}{4} \times \frac{4}{3}$$

$= 1$

$\therefore$ PDF

2 marks awarded for showing this

AND need to show $f(x) \geq 0$

NO ONE DID!

$f(x) = f(4) = 0$

$f(x) > 0 \quad 2 < x < 4$

$f(x) \geq 0 \quad 2 \leq x \leq 4$

All to note this BY the HSC

ii) Mode
- highest pt on $f(x)$ in domain

method one

$$f(x) = \frac{3}{4}(x-2)(4-x)$$

$$= \frac{3}{4}(4x - x^2 - 8 + 2x)$$

$$= \frac{3}{4}(-x^2 + 6x - 8)$$

Axis of Sym $= \frac{-b}{2a}$

$= \frac{-6}{-2}$

$x = 3$

concave DWN parabola!!

∴ Mode is 3.

OR use calculus

method Two

$$f(x) = \frac{3}{4}(-x^2 + 6x - 8)$$

$$f'(x) = \frac{3}{4}(-2x + 6)$$

$$\frac{3}{4}(-2x + 6) = 0$$

$$-2x = -6$$

$$x = 3$$

$$f''(x) = -\frac{3}{2} < 0 \quad \text{max.}$$

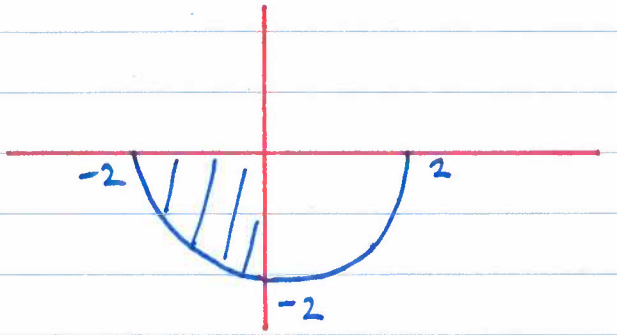
* not well done

* many found the y value not x value

* some confused with 'median' process

c) i) $y = -\sqrt{4 - x^2}$

semi circle !!



ii) Trapezoidal Rule

$$A = \frac{h}{2}\left[f(a) + 2(f.....) + f(b)\right]$$

$$= \frac{0.5}{2}\left[0 + 2(1.3 + 1.7 + 1.9) + 2\right]$$

$\doteq$ 2.95 units^2

iii) More Sub intervals or function values req'd
- this is how to make trapezoidal rule more effective!

Learn Your Basic Graphs!!

* Atrocious!!

* ✓ semi circle
* ✓ shaded LHS
- failure by many to see it was a semi circle!!

* too many weren't sure how to deal with negatives! IGNORE them!!

* done ok by most!

d) a few diff options!!

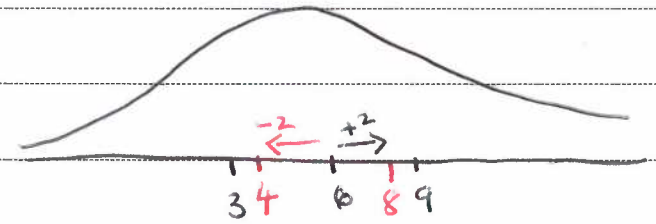
Most Popular	Also	Also
$a = \frac{1}{2}$	$a = -\frac{1}{2}$	$a = \frac{1}{2}$
$b = \frac{5\pi}{4}$	$b = -\frac{3\pi}{4}$	$b = -\frac{3\pi}{4}$
$k = 2$	$k = 2$	$k = -2$
$c = 1$	$c = 1$	$c = 1$

* Not to badly done!!

SECTION C — 16 MARKS.

Marker's Comments.

(a) (i) $P(X>6)$ $\mu = 6, \sigma^2 = 9$ $\therefore$ S.D $\sigma = 3$



It's 50-50 from the median

(ii) $P(X)$ $P(X>6) = 50\%$ or 0.5. ✓

(ii) $P(X>8) = P(z<b) = P(x<4)$ by symmetry

from above : $x = 4$ ✓ Not many realised that you had to find z

$\therefore z = \frac{x-\mu}{\sigma} = \frac{4-6}{3} = -\frac{2}{3}$

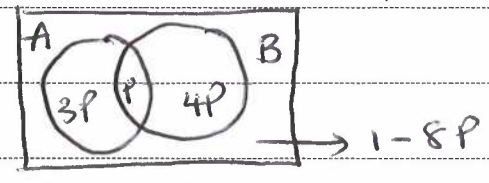
$\therefore b = -\frac{2}{3}$ ✓

(b) (i) $P(A) = ?$ $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$\therefore P(A) = \frac{P(A \cap B)}{P(B|A)} = \frac{P}{1/4}$

$= 4P$ ✓

(ii) $P(A' \cap B')$



✓✓

$P(A) = 4P$ $\therefore P(B) = 5P$

Not well done!

$\therefore P(A' \cap B') = 1 - 8P$

OR $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - 8P$.

(iii) $0 \leq 8P \leq \frac{1}{5}$ $P \neq 0$

$\therefore 0 < P \leq \frac{1}{40}$ or $P = (0, \frac{1}{40}]$ ✓✓

You may ask for an extra Writing Booklet if you need more space to answer this question.

Section C (cont.) 16 MARKS

Marker's Comments.

(c) (i) Accepted Gradian as 0.8 or 0.75

Where appropriate working was shown

$(65,65)$ $(15,25)$ $m = \frac{65-25}{65-12} = 0.8$ ✓✓

OR $(60,60)$ $(40,45)$ $m = \frac{60-45}{60-40} = 0.75$

(ii) $y = mx+b$ using appropriate points

$y = 0.8x+13$ OR $y = 0.75+15$ ✓

(iii) $y = 0.8 \times 80 + 13$

$y = 77k$

$= \$77000$

$y = 0.75 \times 80 + 15$

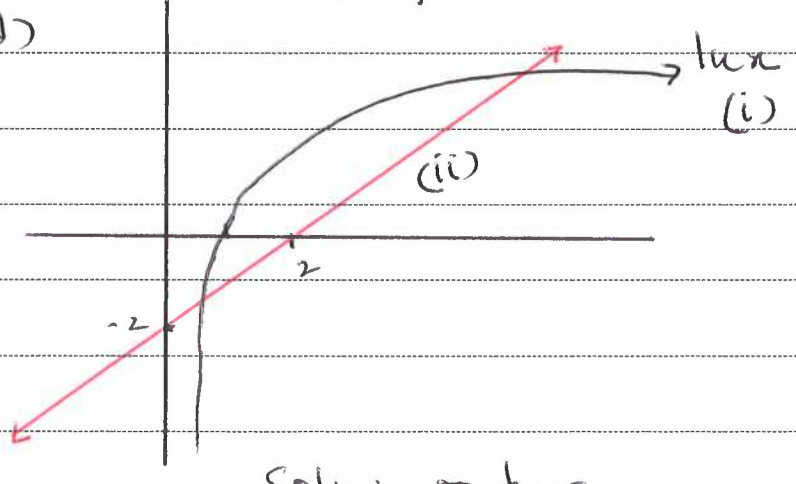
$= 75k$

$= \$75000$ ✓

(iv) With age Salary increase is not true for an 80 year old. or

Cannot extrapolate this equation. ✓

(d)



Soln: ~~x~~ two

Sketch ~~the~~ lux graph

Solutions had to be seen on the graph for 2 Marks.

* Just stating 2 solutions without a graph no marks.

Overall this question was well answered.

You may ask for an extra Writing Booklet if you need more space to answer this question.

2021 2U ADV FINAL TASK

SECTION D

Comments

(a) (i) $M = Ae^{kt}$ $\quad M = \frac{1}{4}A,\ t = 4$

$\frac{A}{4} = Ae^{k4}$ ✓

$\frac{1}{4} = e^{k4}$

$\ln(\frac{1}{4}) = 4k$

$\ln(4^{-1}) = 4k$ ✓

$-\ln(4) = 4k$

$\therefore k = -\frac{1}{4}\ln(4)$.

Can't use Physics formulae "show that" in Part (i) means the examiner must be able to follow every step.

- Many poor attempts at very straight-forward questions
- Use the value for k given in (i) for (ii)

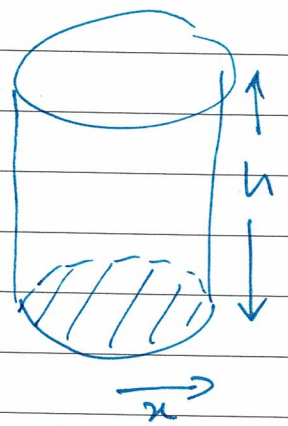
(ii) $M = Ae^{-\frac{1}{4}\ln(4)t}$ $\quad M = 4,\ t = 6$

$4 = Ae^{-\frac{6}{4}\ln(4)}$ ✓

$\therefore A = \dfrac{4}{e^{-\frac{3}{2}\ln(4)}}$

$= 32$ ✓

(b) (i)



$\text{COST (BASE)} = a \times \pi x^2$

$\text{COST (CURVE)} = b \times 2\pi x h$

It is an open cylinder.

"each square metre of the base costs"

(Cost Total) $C = a\pi x^2 + 2b\pi x h$ ✓

(ii) $V_{\text{cyl}} = \pi x^2 h$ $\quad$ [→ No h in given answer]

$V = x \times \pi x h$

It is not a cone

$\therefore 2b\pi x h = C - a\pi x^2$ $\quad$ OR $h = \dfrac{C - a\pi x^2}{2b\pi x}$

$\dfrac{\pi x h}{} = \dfrac{C - a\pi x^2}{2b}$ ✓

Please turn over

D(b)(ii) continued

$$\therefore V = \frac{x(c - a\pi x^2)}{2b} \quad \checkmark$$

Substitution

$$V = \frac{x}{2b}(c - a\pi x^2)$$

(iii) $V = \frac{1}{2b}(cx - a\pi x^3)$

$\therefore V' = \frac{1}{2b}(c - 3a\pi x^2)$ ✓

This is by far the easiest method - Product rule next * It is not quotient rule as $2b$ is a constant term w.r.t. x.

for MAX/MIN let $V' = 0$

$\therefore \frac{1}{2b}(c - 3a\pi x^2) = 0$

$c - 3a\pi x^2 = 0$ ✓

$-3a\pi x^2 = -c$

$a\pi x^2 = \frac{c}{3}$ ✓

$\therefore$ Cost of base $= \frac{c}{3}$ as required ✓

Could also use 1st derivative test but values are required (tough arithmetic/algebra). Reason for statement $V'' < 0$ required (not just a coin toss!).

Now, $V'' = \frac{1}{2b}(-6a\pi x)$

< 0 as $a, b, x > 0$ ✓

$\therefore$ Max value when

$a\pi x^2 = \frac{c}{3}$

SECTION E.

a) i) After month....

$A_1 = 1000(1.0025)$

$A_2 = (A_1 + 1000)(1.0025)$

$= 1000(1.0025)^2 + 1000(1.0025)$ ← 1mk

Well done

$\therefore A^n = 1000(1.0025)^n + \dots + 1000(1.0025)$

$= 1000[1.0025^n + 1.0025^{n-1} + \dots + 1.0025]$

$= 1000\left[1.0025 + 1.0025^2 + \dots + 1.0025^n\right]$

$= 1000\left[\frac{1.0025(1.0025^n - 1)}{1.0025 - 1}\right]$ ← 1mk

$= 401000(1.0025^n - 1)$

ii) let $n = 12$

$A_{12} = 401000(1.0025^{12} - 1)$

$= \$12197$ ← 1mk

Well done

iii) $60000 = 401000(1.0025^n - 1)$

$0.1496 = 1.0025^n - 1$

$1.0025^n = 1.1496$

$n = 55.8$

$\therefore$ 56 DEPOSITS ← 1mk

Well done

Working required

N) TOTAL = A_1 + A_2

$A_1 = 12197(1.0025)^{24}$

$= \$12950.25$ ← 1mk

$A_2 = \dfrac{1500(1.0025(1.0025^{24}-1)}{1.0025-1}$

$= \$37146.80$ ← 1mk

$\therefore$ TOTAL = \$50097

$\therefore$ No !!

Poorly done
Too many ignored A_1. Assumed it earned No Interest

b) i) When $x = \frac{\pi}{3}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\frac{1}{2}\tan\frac{\pi}{3} = \frac{1}{2}(\sqrt{3})$

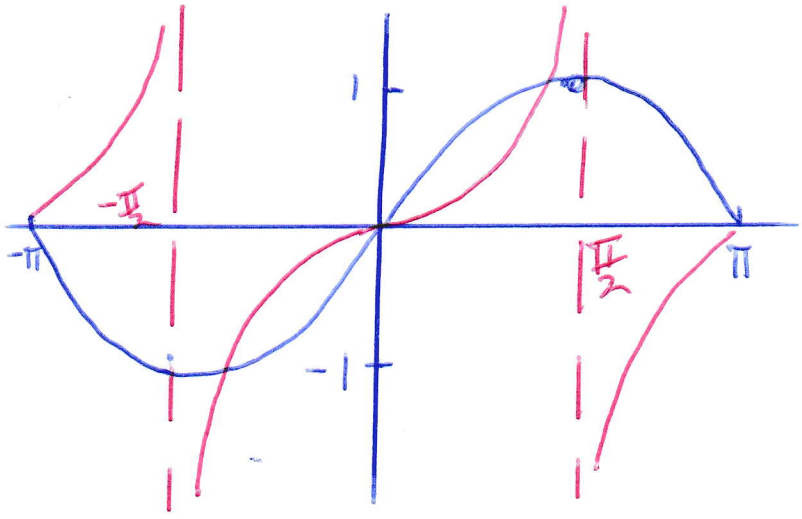
$= \frac{\sqrt{3}}{2}$

} ← 1mk

$\therefore$ at $x = \frac{\pi}{3}$ $\sin x = \frac{1}{2}\tan x$

Many went to a lot of trouble to do something so simple

ii)



$y = \sin x$

$y = \frac{1}{2}\tan x$

Well done

1 mk for each FULLY CORRECT graph.

- Some did not use correct domain

iii) $\sin x = \frac{1}{2}\tan x$

$x = -\frac{\pi}{3}, 0, \frac{\pi}{3}$ ←

Too many missed $x=0$

3 soln's = 2 mk

2 solns = 1 mk

iv) $\sin x \leq \frac{1}{2}\tan x$ for

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

$-\frac{\pi}{3} \leq x \leq 0$ ← 1 mk

$\frac{\pi}{3} \leq x < \frac{\pi}{2}$ ← 1 mk

Poorly done